9.5b Alternating Series, Checking Endpoints

Alternating Series Test with remainder

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n \approx a_1 - a_2 + a_3 - a_4 + a_5 \cdots a_n$$

If 1. signs alternate
2. $a_{n+1} < a_n$
3. $a_n \to 0$

Then series converges

$\epsilon < |a_{n+1}|$ error has same sign as a_{n+1}

Feb 7-10:06 PM

Prove the alternating harmonic series is convergent but not absolutely convergent

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots$$

converges

AST ✓1. signs alt.
✓2. $a_{n+1} < a_n$
✓3. $a_n \to 0$

$$\sum_{n=1}^{\infty} \left|(-1)^{n+1} \frac{1}{n}\right| = 1 + \frac{1}{2} + \frac{1}{3} \cdots$$ harmonic, diverges

Feb 7-10:08 PM

Conditional Convergence

$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges

but $\sum_{n=1}^{\infty} \left|(-1)^{n+1} a_n\right|$ diverges

Feb 7-10:09 PM

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots \qquad \epsilon < \frac{1}{6}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

$x = 1$

Jan 26-9:50 AM

Find the interval of convergence for the following series. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{x^{2n}}{2n}$$

ratio test

① ratio test

$$\lim_{n\to\infty}\left|\frac{x^{2n+2}}{2n+2}\cdot\frac{2n}{x^{2n}}\right| = \lim_{n\to\infty}\left|x^2\,\frac{2n}{2n+2}\right| = x^2 < 1$$

$\sqrt{x^2} < \sqrt{1}$

$|x| < 1$

$-1 < x < 1$

L

$\boxed{-1 \le x \le 1}$

② $x=1$ endpts

$$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{2n} = \frac{1}{2}-\frac{1}{4}+\frac{1}{6}\cdots$$ converges by AST

③ $x=-1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{2n}}{2n} = \frac{1}{2}-\frac{1}{4}+\frac{1}{6}\cdots$$ converges #AST

Feb 7-10:09 PM

$$\sum_{n=0}^{\infty}\frac{(10x)^n}{n!}$$

$$\lim_{n\to\infty}\left|\frac{(10x)^{n+1}}{(n+1)!}\cdot\frac{n!}{(10x)^n}\right| = \lim_{n\to\infty}\left|\frac{10x}{n+1}\right| = \overset{L}{0}$$

for all x

$L<1$

ioc = all x

$(-\infty,\infty)$

Feb 7-10:13 PM

$$\sum_{n=1}^{\infty}\frac{(x-3)^n}{2n}$$

$$\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{2n+2}\cdot\frac{2n}{(x-3)^n}\right| = \lim_{n\to\infty}\left|(x-3)\frac{2n}{2n+2}\right| = \overset{L}{|x-3|}$$

$|x-3| < 1$

$-1 < x-3 < 1$

$\boxed{2 \le x < 4}$

$x=4$ $$\sum_{n=1}^{\infty}\frac{1}{2n} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$$ harmonic, diverges

$x=2$ $$\sum_{n=1}^{\infty}\frac{(-1)^n}{2n} = -\frac{1}{2}+\frac{1}{4}-\frac{1}{6}\cdots$$ Converges by AST

Feb 7-10:15 PM